CMPSC 250
Analysis of algorithms

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Lecture 05: Finishing up Josephus problem
Problem Definition: A group of n people are standing in a circle, numbered consecutively clockwise from 1 to n. Starting with person no. 2, we remove every other person, proceeding clockwise. For example, if n = 6, the people are removed in the order 2, 4, 6, 3, 1, and the last person remaining is no. 5. Let \( j(n) \) denote the last person remaining. Find some simple way to compute \( j(n) \) for any positive integer \( n > 1 \).
Finishing up Josephus problem (2)

**Algorithm** Josephus(n, m):

for i ← 1 to n do
    circle.enqueue(i)
while circle.size() > 1 do
    for j ← 1 to m do
        circle.enqueue(circle.dequeue())
    circle.dequeue()
return circle.front()
Finishing up Josephus problem (3)

- Let us say there are N no of people in the circle and \( W(N) \) defines the winning position for Nth person.

Deriving Mathematically and finding the pattern for some sample inputs:

- \( N = 2^x + y \)
- \( W(N) = 2y + 1 \)
Finishing up Josephus problem (4)

- Using binary form representation.
  Remove first bit and append to the last bit.
- For example: if $N = 5$ then $W(N) = 3$
  5 in binary is 101
  Removing first bit and append to last bit give us:
  011 - which is 3 in binary.
In Class Activity:
Reading Exercise:

- Section 1.3 Pg 128 - 131, 150 -153 in textbook.
- Practice Dijkstra’s two stack algorithm on page 129.
Questions