Last Time

• Graphs
  – Definitions
  – Path finding
  – Minimum spanning trees
  – Shortest paths
Why Does Sorting Matter?

- “About a quarter of all computer cycles are spent sorting.” – Donald Knuth, 1973
  - Bank account transactions
  - Search engine results
  - Scientific computations – astrophysics, molecular dynamics, weather prediction, linguistics
- Suitable “prototype problem” – easily modeled and has good mathematical properties.
- The first step towards organizing and evaluating data is often to sort it.
Selection Sort

• Find the smallest item in the array; put it first.
• Find the next smallest item; put it second.
• Repeat until you’ve reached the last item in the input array.

```
for (int i = 0; i < N; i++) {
    int min = i;
    for (int j = i+1; j < N; j++) {
        if (a[j] < a[min]) {
            min = j;
        } //if
    } //for
    exch(a, i, min);
} //for
```
Selection Sort Visual

Trace of selection sort (array contents just after each exchange)
Selection Sort Evaluation

• Compares
  – When $i = 1$, we compare it against the other $(n - 1)$ entries.
  – When $i = 2$, we compare it against the remaining $(n - 2)$ entries.
  – $(n - 1) + (n - 2) + \cdots + 2 + 1 = \sim \frac{n^2}{2}$ compares

• `exch()` exchanges
  – For each $i$ value, we do one exchange, swapping $a[i]$ with $a[min]$.
  – $1 + 1 + 1 + \cdots + 1 = n$ exchanges
Selection Sort Evaluation

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<th>i</th>
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Trace of selection sort (array contents just after each exchange)

Entries in black are examined to find the minimum.
Entries in red are \( a[\text{min}] \).
Entries in gray are in final position.
Selection Sort Evaluation

• Run time is insensitive to input.
  – Finding the smallest item on iteration $i$ does not give any information about the location of the smallest item in iteration $(i + 1)$.
  – Therefore, worst case = average case.

• Data movement is minimal.
  – Number of exchanges is linear w.r.t. array size.
  – No other sorting algorithm that we will consider has this property.
Insertion Sort

• Look at the current $a[i]$.
• Place it appropriately between items $a[0]$ to $a[i-1]$, moving it left until it shouldn’t be moved further.
• Repeat until you’ve reached the last item in the input array.

```c
for (int i = 1; i < N; i++) {
    for (int j = i; j > 0 && (a[j] < a[j-1]); j--) {
        exch(a, j, j-1);
    } //for
} //for
```
### Insertion Sort Visual

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<tr>
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*Trace of insertion sort (array contents just after each insertion)*

- Entries in gray do not move.
- Entry in red is a[i].
- Entries in black moved one position right for insertion.
Insertion Sort Evaluation

• Compares
  – When $i = 1$, we compare it against a maximum of 1 previous entry.
  – When $i = 2$, we compare it against a maximum of 2 previous entries.
  – On average, assume we’re moving the new value halfway to the left.
  – $[1 + 2 + \cdots + (n - 2) + (n - 1)]/2 = \sim \frac{n^2}{4}$ compares

• `exch()` exchanges
  – Since `exch()` is called in a loop limited by `less()` calls, the count is identical, $\sim \frac{n^2}{4}$ exchanges
Insertion Sort Evaluation

• Now we have a worst case and a best case to consider:
  
  – **Worst case:** We need to move every letter the whole way to the left.
    
    • $1 + 2 + \cdots + (n - 2) + (n - 1) = \sim \frac{n^2}{2}$ compares and exchanges.
  
  – **Best case:** We don’t need to move any letters – the array is already sorted, or all of the keys are identical.
    
    • $1 + 1 + 1 + \cdots + 1 = n - 1$ compares and 0 exchanges.
Insertion Sort Evaluation

Trace of insertion sort (array contents just after each insertion)

- **Entries in Gray**: Do not move.
- **Entry in Red**: This is $a[j]$.
- **Entries in Black**: Moved one position right for insertion.
Insertion Sort Evaluation

• Run time and data movement are both sensitive to input.
  – The initial positions of the items has a significant impact on the run time of the algorithm, as well as how far each data item needs to move.
  – Insertion Sort works quite efficiently on data that is already almost sorted and just needs a few tweaks:
    • A small array appended to a large sorted array.
    • An array that was sorted and had a few values update.
Comparing Sorting Algorithms

- So which is faster, Insertion or Selection Sort?
  - It depends...
Any Questions?