Last Time

- Nondeterminism!
  - What is it?
  - Why do we care?
  - DFA and NFA equivalency
  - NFAs to prove regular language properties
Regular Expressions

• Regular Expression – an expression that describes a regular language.

\[(0 \cup 1)0^* = \text{strings beginning with a 0 or 1, followed by any number of 0s}\]

Expands out to \((\{0\} \cup \{1\}) \circ \{0\}^*\)
Regular Expressions

• R is a regular expression if R is:
  – \( a \), for some \( a \) in the alphabet \( \Sigma \),
  – \( \varepsilon \),
  – \( \emptyset \),
  – \((R_1 \cup R_2)\), where \( R_1 \) and \( R_2 \) are regular expressions,
  – \((R_1 \circ R_2)\), where \( R_1 \) and \( R_2 \) are regular expressions,
  – \((R_1^*)\), where \( R_1 \) is a regular expression.
Some Examples

• $0^*10^*$
  – \{w \mid w \text{ contains a single 1}\}

• $\Sigma^*1\Sigma^*$
  – \{w \mid w \text{ has at least one 1}\}

• $\Sigma^*001\Sigma^*$
  – \{w \mid w \text{ contains the substring 001}\}

• $(\Sigma\Sigma)^*$
  – \{w \mid w \text{ is a string of even length}\}
Some Examples

• \(0\Sigma^*0 \cup 1\Sigma^*1 \cup 0 \cup 1\)
  
  \(\{w \mid w \text{ starts and ends with the same symbol}\}\)

• \((0 \cup \varepsilon)1^*\)
  
  \(\{w \mid w \text{ is either } 0 \text{ or } \varepsilon, \text{ followed by } 1^*\}\)

• \((0 \cup \varepsilon)(1 \cup \varepsilon)\)
  
  \(\{\varepsilon, 0, 1, 01\}\)
Two Annoying Examples

• $1^*\emptyset$
  – Concatenating the empty set to any set yields the empty set, so $1^*\emptyset = \emptyset$

• $\emptyset^*$
  – The star operation puts together any number of strings from the languages to get a string in the result. If the language is empty, the star operation can put together 0 strings, giving only the empty string. Thus, $\emptyset^* = \varepsilon$. 
Now, Some Identities

- \( R \cup \emptyset = R \)
  - Adding the empty language to any other language will not change it.

- \( R \circ \varepsilon = R \)
  - Appending the empty string to any string will not change it.
Now, Some Trickery

• $R \cup \varepsilon = R$?
  
  – Consider $R = 0$. Then $L(R) = \{0\}$, but $L(R \cup \varepsilon) = \{0, \varepsilon\}$.

• $R \circ \emptyset = R$?
  
  – Consider $R = 0$. Then $L(R) = \{0\}$, but $L(R \circ \emptyset) = \emptyset$. 
Why Are We Talking About Regular Expressions Anyway?

• Theorem: A language is regular iff some regular expression describes it.
  
  – **Forward Direction:** If a language is described by a regular expression, then it is regular.
  
  – **Proof Idea:** Assume we have a regular expression $R$ describing some language $A$. We can convert $R$ into an NFA recognizing $A$. NFAs are equivalent to DFAs, and if a language is recognized by a DFA, then it is regular.
OK, Let’s Convert R into an NFA...

• We have to consider all 6 cases of the regular expression definition...
  1. $R = a$, for some $a$ in the alphabet $\Sigma$
  2. $R = \varepsilon$,
  3. $R = \emptyset$,
  4. $R = (R_1 \cup R_2)$
  5. $R = (R_1 \circ R_2)$
  6. $R = (R_1^*)$
What About the Other Half of the Proof?

• Theorem: A language is regular iff some regular expression describes it.
  
  – **Backward Direction:** If a language is regular, then it is described by a regular expression.
  
  – **Proof Idea:** If a language $A$ is regular, then it is accepted by a DFA. Therefore, if we can show a way to convert a DFA to a regular expression, then the language recognized by that DFA is also recognized by that regular expression.
  
  – (This is a really messy proof that I won’t subject you to...)
Let’s Turn Another Regular Expression into an NFA now

![Diagram of an NFA](image)
How About Turning a DFA into a Regular Expression?
Any Questions?

HOMEWORK (due 9/16)
1.16a, 1.17a, 1.18abdm, 1.19a, 1.20abcde, 1.21a
Image Credits

• Slide 13: Regular Expression -> DFA (Sipser textbook, 2\textsuperscript{nd} edition)
• Slides 14: DFA -> Regular Expression (Sipser textbook, 2\textsuperscript{nd} edition)