A Comparison of Different Automated Market-Maker Strategies

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Abstract. Financial markets such as stock exchanges and electronic prediction markets frequently use the services of an entity called the market-maker to ensure that the market's traders can make their transactions. Recently, several strategies that can be used by market-makers to control market trading prices have been proposed by various researchers. A detailed comparison of these market maker strategies using real trading data extracted from financial markets is essential to understanding the relative merits and requirements of the different market-maker strategies. We address this aspect of market-maker strategies by empirically comparing different strategies with data obtained from the NASDAQ market. Our results show that a reinforcement learning-based strategy performs well in maintaining low spread as well as in obtaining high utilities, whereas other strategies only succeed in either maintaining low spread or outperforming others in utilities. [1]

Keywords: Market-maker strategy, electronic financial markets, market-maker simulation.

1 Introduction

Over the past few years, the rapid growth and success of automated techniques for e-commerce have resulted in their wide adoption in various domains beyond traditional B2B and B2C commodity markets. For example, in financial markets human traders are being replaced by automated agents that efficiently buy and sell financial securities. Currently many modern exchanges, such as NYSE, NASDAQ, and Toronto Stock Exchange as well as electronic prediction markets, such as tradesports.com, use such automated agents called *market-makers* to regulate prices and quantities of securities or stocks traded by the market's participants.

A market-maker holds a certain number of securities in its inventory with the purpose of being able to sell them to an interested buyer, or to buy securities from a seller selling securities in the market. A market can have either a single market-maker or multiple market-makers that compete with each other. The

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fundamental role of a market-maker is to bring buyers and sellers together so that trading can occur in an efficient and fair manner. The main advantage of automated market-makers is its ability to maintain the liquidity [2] in the market [6]. The liquidity in turn reduces the trading cost of the market's participants [1]. Additionally, market-makers can help to smooth price fluctuations due to spurious supplies or demands [15]. Appropriately designed automated market-makers do not have an incentive to engage in the market manipulation [16].

As the role of the market-makers grows, the need for better understanding of the impact of the market-makers in the market increases as well. In this paper we use a model of a financial market with multiple market-makers to study the potential impact of widespread automated market-maker usage on market dynamics. We investigate four different strategies for automated market-making in financial markets - a myopically optimizing strategy, a reinforcement learning strategy, a market scoring rule and a utility maximizing strategy - with the goal of testing the existing strategies against each other and examining their strengths and weaknesses. Our experiments reveal that the myopically optimizing and market scoring rule-based strategies perform well in maintaining low spread and smooth market price, however they fall short in maximizing utilities as compared to other strategies. On the other hand, the utility maximizing strategy with different risk attributes performs very well in obtaining high utilities, although it fails in maintaining low and consistent spread. Consequently, the market price tends to fluctuate significantly. Finally, the reinforcement learning strategy fulfills its tasks of both controlling the spread and maximizing utility.

2 Related Work

The automation of a market-makers's functions was suggested more than three decades ago [2]. Previously, several theoretical approaches, albeit with certain simplifying assumptions, have been proposed to understand the effects of market-makers on financial markets. Garman [7] describes a model with a single, monopolistic market-maker, who sets prices, receives orders and clears trades and tries to maximize expected profit per unit time. Such market-maker fails when it runs out of inventory or cash. In [14], the authors study the optimal behavior of a single market-maker who gets a stochastic demand and tries to maximize its expected utility of final wealth, which depends on the profit it receives from trading. Glosten and Milgrom [10] investigate the market-making model with asymmetric information. Das [6] empirically studies different market-making strategies and concludes that a heuristic strategy that adds a random value to zero-profit market-makers improves the profits in the markets. Gu [11] explores changing the market-maker behavior by estimating the market-maker's profitability under different parameters. The results show that a profit-maximizing market-maker's objectives may not align with price variance minimization, which can be one of the qualities of an orderly market. Westerhoff [17] also explores the

[2] A market is said to be liquid if traders can buy or sell large quantities of a security without causing large changes in the market price. Liquidity is a valuable characteristic of a market because it enables high volume trades.

impact of inventory restrictions in a setup with an implied market-maker. The market-maker price adjustment reactions differ depending on the current inventory position along with current excess demands. The market-maker is assumed to make greater price adjustments when these two variables are of the same sign. Market-making has also been adopted as a test-bed for machine learning techniques [15] with a goal to demonstrate the general effectiveness of a learning algorithm.Also, empirical work has demonstrated the limitations of hard-coding market-making rules into an algorithm [12]. In [16], the primary goal is to optimally change the spread over the next iteration instead of finding the best model for past transactions.

In the past, several market-maker strategies have been proposed and there have been a few studies on the market-maker's effect on the market. Most of these past studies focus on a market with a single market-maker or a market with multiple market-makers of the same strategy. However, there does not exist a study comparing the effect of different market-maker strategies in a market with multiple market-makers. In this paper, we attempt to address this deficit by providing experimental results using data from real security markets to examine the behavior of a market with multiple market-makers that employ different competing strategies. We also analyze the effect of each market-making strategy and the combinations of strategies on the market price dynamics.

3 Model

We have adapted the well-known Glosten and Milgrom [6, 10] model of financial markets to a multi-agent framework of a financial market with multiple electronic market-makers. In our model, each human trader is modeled as a software agent called a trading agent that embodies the behavior of a human trader.

The market consists of N traders and M market-makers who buy and sell securities or stocks, where $N >> M$. Each *trading episode* e consists of T *trading periods*. Each stock s, has a true or fundamental value $V_{s,e}$ at trading episode e. That is, there is some exogenous process that determines the value of the stock. The true price of a stock is different from the market price at which the stock gets traded. The market price of a stock is determined by the interaction between the market-makers and the traders. The stock's true price $V_{s,e}$ gets updated during each trading episode with some probability $\pi_{s,e+1}$ according to the following equation [5, **?**]:

$$V_{s,e+1} = V_{s,e} + jump, \tag{1}$$

where $jump$ is a parameter sampled from a normal distribution with mean $\mu_{V s,e}$ and variance $\delta_{V_{s,e}}$. The jump in the true value of the stock can be positive or negative and usually corresponds to some new information about the stock arriving in the market from external sources. The volatility of the stock value is influenced by the value of the standard deviation of the jump and the probability that the jump will occur.

Market-maker m's bid(buy) price of the stock s at trading period t, $p^{sell}_{m,s,t}$, is the price the market-maker charges traders for buying 1 share of stock s.

The market sell price of the stock s at trading period t, $P^{sell}_{s,t}$, is the price that the market-maker pays to traders for buying 1 share of the stock s. The market bid(buy) price $P^{buy}_{s,t}$ at trading period t for stock s is the maximum of the market-makers' bid prices. The market ask(sell) price $P^{sell}_{s,t}$ at trading period t for stock s is the minimum of the market-makers' ask prices.

The different parameters used in our financial market model to define the market characteristics and specify the market-makers and trading agents behavior are shown in Table 1 and described below.

Market	**Parameters**
e	Trading episode
t	Trading period
N	Number of traders in the market
M	Number of market-makers in the market
S	Number of stocks
$V_{s,e}$	True value of stock s during trading episode e
$\pi s, e$	Probability that the jump in the true value of stock s occurs during trading episode e
$P^{sell}_{s,t}$	Market sell price of the stock s at trading period t
$P^{buy}_{s,t}$	Market buy price of the stock s at trading period t
Market-Maker Agent	**Parameters**
$p^{sell}_{m,s,t}$	Market-maker m's sell price of the stock s at trading period t
$p^{buy}_{m,s,t}$	Market-maker m's buy price of the stock s at trading period t
θ_m	Risk coefficient of the market-maker agent m
u_m	Market-maker m's utility

Table 1. Parameters used in our model.

3.1 Trader Behavior

When a trader enters the market, it is randomly assigned to some market-maker. At each trading period t, traders place a buy or sell order, or no order at all, based on the buy or sell price of the stock given by the market-maker. Each trader n has a valuation for each stock s, $W_{n,s}$ sampled from a normal distribution. If $W_{n,s} > P^{sell}_{s,t}$, the trader buys one unit of the stock s, if $W_{n,s} < P^{buy}_{s,t}$, the trader sells one unit of the stock s, and if $P^{buy}_{s,t} \leq W_{n,s} \leq P^{sell}_{s,t}$, the trader holds the stock.

3.2 Market-maker Behavior

At each trading period t, each market-maker sets the bid and ask prices for each stock according to some algorithm. The difference between the bid and ask prices is called the stock's *spread*. Market-makers use this spread of a stock to ameliorate their risks of holding a considerable quantity of the stock. Market-makers execute the buy or sell stock orders from the traders immediately. A market-maker does not know the true value of a stock, but it receives a noisy signal about the jump in the true value of the stock, $jump + \tilde{N}(0, \delta_m)$, where $jump$ is the actual jump that has occurred and $\tilde{N}(0, \delta_m)$ represents a sample from a normal distribution with mean 0 and variance δ_m^2. Figure 1 shows the operations of the market-maker in a market.

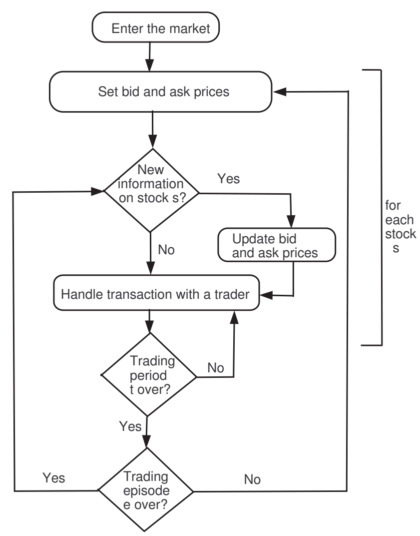

Fig. 1. A flowchart showing the operation of the market-maker agents in the market.

Before we present our experimental results, we first briefly review the market-maker algorithms that we use for our comparisons.

A Myopically Optimizing Market-Maker A Myopically Optimizing Market-Maker uses an algorithm developed by Das in [6]. The key aspect of the algorithm is that the market-maker uses the information conveyed in trades to update its beliefs about the true value of the stock, and then it sets buy/ask prices based on these beliefs. The market-maker maintains a probability density estimate over the true price of the stock. There are two key steps involved in the market-making algorithm. The first is the computation of bid and ask prices given a probability density estimate over the true price of the stock, and the second is the updating of the density estimate given the information implied in trades. This market-maker optimizes myopically, setting the prices that give the highest expected profit at each trading period. That is, the optimal buy price is the price that maximizes the expression $E(profit_s | p^{buy}_{m,s,t} = x)$, and the optimal sell price is the price that maximizes the expression $E(profit_b | p^{sell}_{m,s,t} = x)$, where

$profit_s(profit_b)$ is the profit from a marginal sell(buy) order being received. The market-maker uses the Bayesian updating method described in [5] to update its density estimates. All of the points in the density estimate are updated based on whether a buy order, sell order, or no order was received. The density estimate is initialized to be normal.

Reinforcement Learning Market-Maker Chan and Shelton [3] have modeled the market-making problem in the framework of reinforcement learning. They have used Markov decision process (MDP) to model reinforcement learning of a market-maker. A state is defined as $s_t = (inv_{m,t}, imb_t, qlt_t)$, where $inv_{m,t}$ is the market-maker m's inventory level, imb_t is the order imbalance, and qlt_t is the market quality at trading period t. The inventory level is the market-maker's current holding of the stock. The order imbalance is calculated as the sum of the buy order sizes minus the sum of the sell order sizes during a certain period of time t. Market quality measures are the bid-ask spread and price continuity (the amount of price change in subsequent trades). Given the states of the market, the market-maker reacts by adjusting the bid/ask prices and trading with incoming orders. The action vector for market-maker m is defined as $a_{m,t} = (p^{buy}_{m,t}, p^{sell}_{m,t})$.

The market-makers can obtain the optimal strategy by maximizing the profit, by minimizing the inventory risk, or by maximizing market qualities. Thus, the reward at each time step depends on the profit received, the change of inventory, and the market quality measures. This strategy assumes the risk-neutrality of the market-maker.

Utility Maximizing Market-Maker with Risk attributes Previous research [9, 19] has shown that by considering risk-taking and risk-averse behaviors of the human traders, the behavior of the market can be improved. We set out to see if the incorporating the risk behavior of the market-makers can improve the financial market performance. Following [9], we adopt a constant relative risk averse (CRRA) utility function $\widetilde{u}_{m,t}$ for market-maker m with a relative risk aversion coefficient. CRRA utility functions have been widely used to model risk behaviors. Relative risk aversion coefficient, θ_m, is used to classify market-maker m's risk levels as follows. If $\theta_m > 0$, the market-maker m is risk-averse, if $\theta_m = 0$, the market-maker m is risk-neutral, and if $\theta_m < 0$, the agent m is risk-seeking. Unless otherwise specified, the market-makers' risk coefficients are normally distributed in our simulations. Following the trading agent utility model in [9], during each trading period t market-maker m uses its instantaneous utility $\acute{u}_{m,t}$ and its risk-taking coefficient to calculate its modified instantaneous utility for that trading period, using Equation 2.

$$\widetilde{u}_{m,t}(\acute{u}_{m,t}, \theta_m) = \begin{cases} \frac{\acute{u}_{m,t}^{1-\theta_m}}{1-\theta_m} & \text{, if } \theta_m \neq 1; \\ ln(\acute{u}_{m,t}) & \text{, if } \theta_m = 1. \end{cases} \tag{2}$$

These market-maker agents are utility maximizers, that is they update prices so that their overall utility is maximized.

LMSR Market-Maker Hanson invented a market-maker for the use in prediction market applications called the logarithmic market scoring rule (LMSR) market-maker [13]. We have used Chen and Pennock's formulation of Hanson's (LMSR) market-maker [4]. Let $\overline{q} = (q_1, q_2 ... q_N)$ be the vector specifying quantities of stocks held by the different trading agents in the market. The total cost incurred by the trading agents for purchasing these stocks is calculated by the market-maker using a cost function $C(\overline{q}) = b \cdot ln(\sum_{j=0}^{|\overline{q}|} = e^{q_j/b})$. The parameter b is determined by the market-maker and it controls the maximum possible amount of money the market-maker can lose as well as the quantity of shares that agents can buy at or near the current price without causing massive price swings. If an agent purchases a quantity δ_q of the security, the market-maker determines the payment the agent has to make as $p^{buy}_{s,m,t} = C(\overline{q} + \delta_q) - C(\overline{q})$. Correspondingly, if the agent sells δ_q quantity of the security, it receives a payment of $p^{sell}_{s,m,t} = C(\overline{q}) - C(\overline{q - \delta_q})$ from the market-maker.

4 Experimental Results

We have compared the four market-maker algorithms described in the previous section through several simulations. The true value for stock s during episode s was obtained from the data of real NASDAQ stock markets. First ten stocks were randomly selected from all the stocks traded on NASDAQ. Then the real data of those ten stocks was downloaded from Yahoo! Finance [20]. We have used open prices of each day to simulate the true value of the stock at the beginning of each trading episode.

Each trading episode consists of 100 trading periods, where each trading period lasts for 0.5*sec*. We simulate the financial market with 100 traders and 3 or 2 market-makers. We show the results of our simulations over 100 trading episodes.

First we want to observe the behavior of the market with market-makers that use the same strategy. After that we perform the pairwise comparison of different market-maker strategies and evaluate the performance of each one in more detail. We report the market price, the spread, and the utility earned by the each type of the market-maker used in our simulations. The market price and the spread evaluate the quality of the market, whereas the utility evaluates the profitability of the strategy employed by the market-maker. In our graphs we show the results for the Yahoo! stock. Figure 2 shows the Yahoo! stock's price obtained from [20]. In our first set of experiments there are 3 market-makers that use the same strategy in the market. Figure 3 shows the simulations of the market with myopically optimizing market-makers. We can see from the spread graph that the myopically optimizing market-maker is sensitive to the price variations in the market. The spread value has large fluctuations following the jump in the true value of the stock. Spread seems to stabilize somewhat until the next jump. Due to large jumps in the spread value, myopically optimizing market-makers are able to keep increasing their utilities. Myopically optimizing market-makers are able to avoid causing big jumps in the market price, which is one of the important functions of the market-makers.

Figure 4 shows the simulations of the market with reinforcement learning market-makers. The utility of the reinforcement learning market-makers is ex-

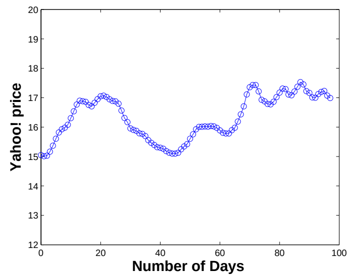

Fig. 2. Yahoo! price data used in our simulations.

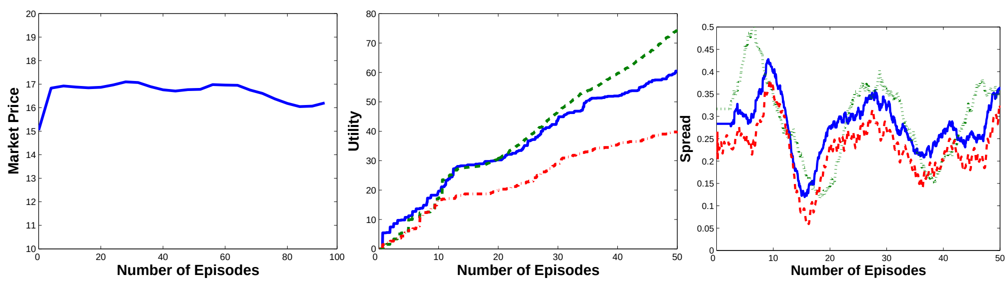

Fig. 3. Myopically Optimizing Market-Makers.

pected to improve with each trading episode. As expected, these market-makers perform very well with respect to utility-maximization. However, the spread value fluctuates somewhat throughout the trading episodes. The market price does not fluctuate a lot throughout the simulation.

Figure 5 shows the simulations of the market with logarithmic market scoring rule (LMSR) market-makers. LMSR market-makers perform very well the function of maintaining an orderly market. That is, the market price is smooth and the spread is steady and consistent. LMSR market-makers do not aggressively maximize their utility, as can be seen from the utility graph in Figure 5.

Figure 6 shows the simulations of the market with utility maximizing market-makers with different risk attributes, i.e. with one risk-taking, risk-neutral, and risk-averse market-maker. We can see that the risk-taking market-maker is able to obtain slightly higher utility than the risk-neutral and risk-averse market-makers. Risk-averse market-maker gets the least utility, but maintains the smallest spread. Risk-taking market-maker does not control the spread value well, as it fluctuates a lot and by large amounts. Also, the market price has more fluctuations with these market-makers than with other types of market-makers.

For our next set of simulations we perform pairwise comparisons of different market-maker strategies. We simulate the market with 2 market-makers, one of

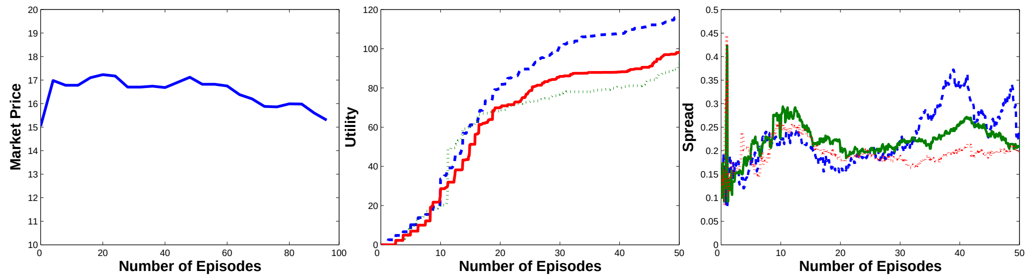

Fig. 4. Reinforcement Learning Market-Makers.

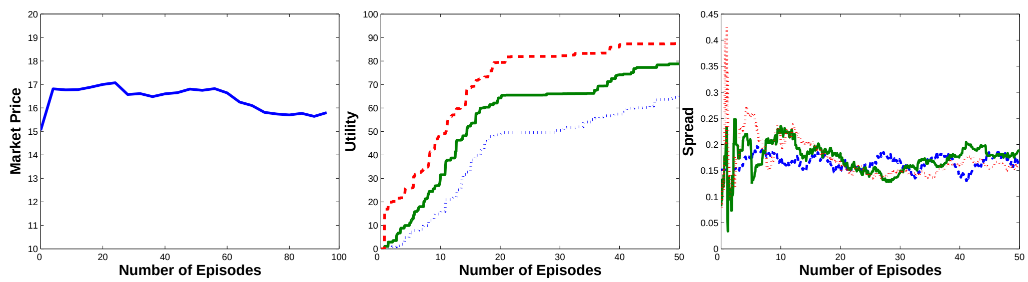

Fig. 5. LMSR Market-Makers.

each type. However, when comparing utility maximizing market-makers with 3 different risk attributes, we use 4 market-makers in the market.

First we compare myopically optimizing market-maker with 3 utility maximizing market-makers, one risk-taking, one risk-neutral, and one risk-averse market-maker. As can be seen from Figure 7, the fluctuations in the market price are pretty significant. We foresee that this is mainly due to presence of utility maximizing market-makers, since their primary function is not the control of the quality of the market, but utility maximization. Although, it is interesting to see that the risk-averse utility maximizing market-maker is able to maintain steady and low spread, and is very compatible in that regard with the myopically optimizing market-maker. Myopically optimizing market-maker also outperforms the risk-averse market-maker in overall utility.

Figure 8 illustrates the market with one myopically optimizing market-maker and one LMSR market-maker. We can see that these market-makers contribute to maintaining smooth market price and close spread values. However, myopically optimizing market-maker outperforms the LMSR market-maker by 40% on average in utility.

In Figure 9 we present the comparison of the myopically optimizing market-maker with reinforcement learning market-maker. Our results show that reinforcement learning market-maker is able to obtain 24% higher utility on average than the myopically optimizing market-maker. However, myopically opti-

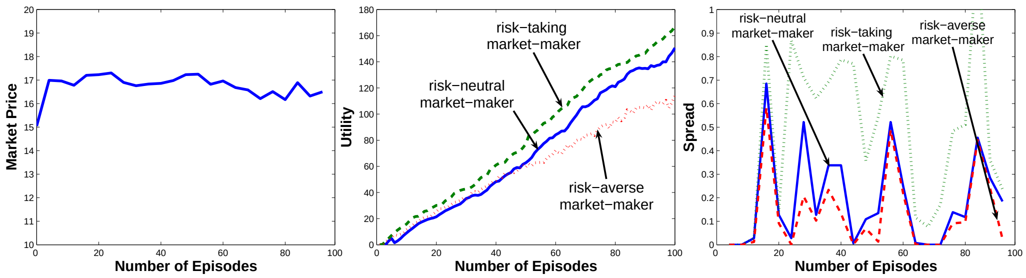

Fig. 6. Utility Maximizing Market-Makers with different risk attributes.

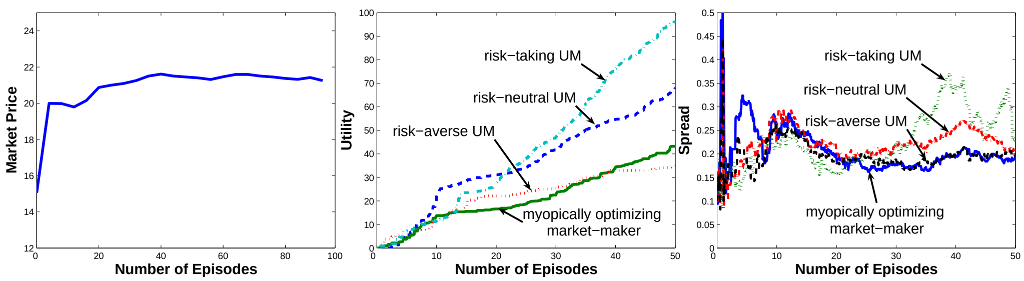

Fig. 7. Myopically Optimizing Market-Maker versus Utility Maximizing Market-Makers with different risk attributes.

mizing market-maker maintain 6.5% less spread on average than the reinforcement learning market-maker. Both market-makers do a good job in maintaining smooth market price and steady spread.

Next we compare the performance of the LMSR market-maker with 3 utility maximizing market-makers, i.e. risk-taking, risk-neutral, and risk-averse market-maker. We can see from Figure 10 that the volatility in the market is significant, with the fluctuations in the market price and large variations in the spread values. All utility maximizing market-makers outperform LMSR market-maker in utility. For example, risk-taking utility maximizing market-maker obtains 49% higher utility than LMSR market-maker. However, LMSR market-maker has 31% lower average spread than the risk-taking market-maker, which has the highest spread.

Figure 11 shows the performance of the reinforcement learning market-maker against the utility maximizing market-maker with different risk attributes. Our results indicate that the reinforcement learning market-maker has lower spread. In particular its average spread is 25%, 22%, and 13% lower than the risk-taking, risk-neutral, and risk-averse utility maximizing market-makers. Also, reinforcement learning market-maker is able to outperform risk-averse market-maker in utility by 11%, but it receives 69% less utility than risk-neutral market-maker, and over 100% less utility than the risk-taking market-maker.

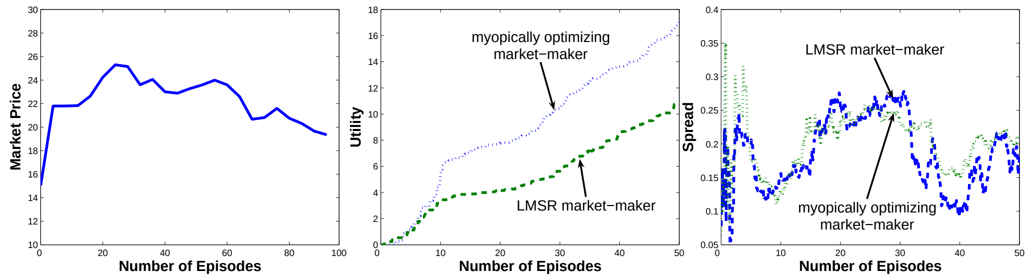

Fig. 8. Myopically Optimizing Market-Maker versus LMSR Market-Maker.

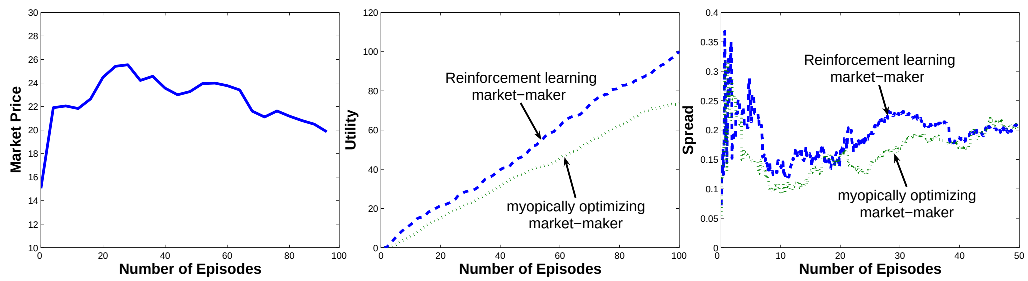

Fig. 9. Myopically Optimizing Market-Maker versus Reinforcement Learning Market-Maker.

Reinforcement learning market-maker performance comparison with LMSR market-maker is shown in Figure 12. The market price is smooth throughout 100 trading episodes. Although reinforcement learning market-maker obtains 54% more utility than the LMSR market-maker, the spread difference between two market-makers is not very significant (8%).

Finally, we simulate the market with all four market-makers as shown in Figure 13. The market price is smooth throughout 100 trading episodes. We observe that the utility maximizing, risk-neutral market-maker outperforms other market-makers in utility. The difference in the utility between the utility maximizing market-maker and LMSR market-maker (which got the least utility) is on average 60%. However, LMSR market-maker is able to maintain the lowest spread in the market. The difference in spread between the LMSR market-maker and the utility maximizing, risk-neutral market-maker is on average 73%. However, the spread difference between reinforcement and myopically optimizing market-makers is not very significant (11%).

5 Discussions and Lessons Learned

In this paper, we have used an agent-based financial market model to analyze the dynamics in the market with multiple market-makers. We investigated the effects

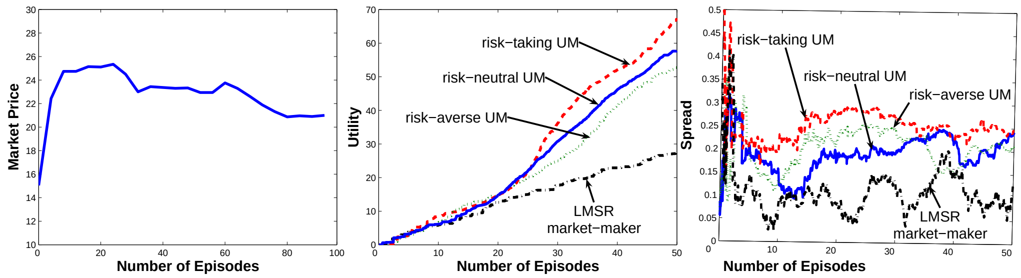

Fig. 10. LMSR Market-Maker versus Utility Maximizing Market-Makers with different risk attributes.

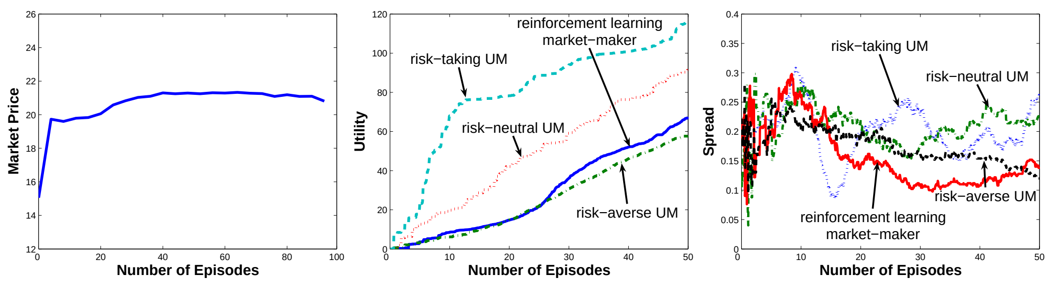

Fig. 11. Reinforcement Learning Market-Maker versus Utility Maximizing Market-Makers with different risk attributes.

of various market-making strategies on the market prices and market-makers' spread and utilities. The difficulty in constructing the market-making strategies comes from the need for the market-maker to balance conflicting objectives of maximizing utility and market quality, that is fine-tuning the tradeoff between utility and market quality.

Our simulation results show that the utility maximizing risk-taking and risk-neutral market-makers outperform all the other types of market-makers in utility, however they lack in maintaining the market quality, i.e. low and continuous spread and smooth market price. Myopically optimizing market-maker performs well with both maintaining good market quality and obtaining high utility. Reinforcement learning market-maker has comparable results when it comes to utility compared to the other market-maker strategies that are designed with a primary goal of maintaining market quality. Reinforcement learning market-makers also do their job of market control very well. LMSR market-maker does not do so good in terms of maximizing its utility, since it is not designed to do that. However, it performs well in maintaining continuously low spread.

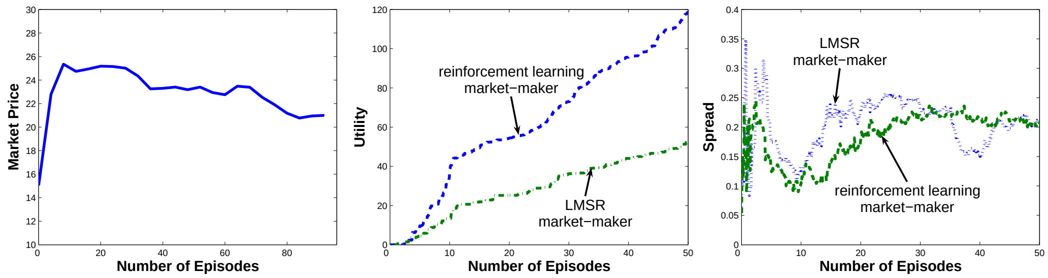

Fig. 12. Reinforcement Learning Market-Maker versus LMSR Market-Maker.

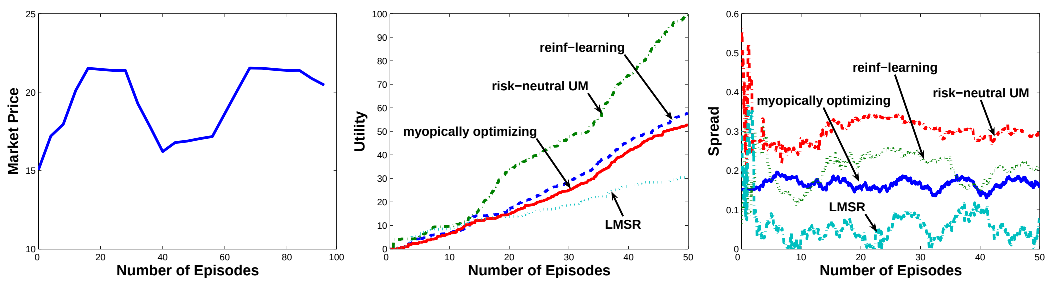

Fig. 13. Market with Reinforcement Learning, Utility Maximizing, Myopically Optimizing and LMSR market-makers.

6 Future work

This is our first step in performing a comparison of multiple market-maker strategies. In the future, we wish to explore different extensions of this work. First of all, we would like to propose and perform comparisons of other market-maker strategies such as using a minimax regret algorithm for price adjustments by the market-maker. Secondly, we would like to study the performance of the market-makers with a more complex behavior, such as dynamically switching strategies based on past performance. This way, a better balance of maintaining market quality and maximizing market-maker utilities may be obtained. And lastly, we would like to add various behavioral attributes to the market-maker model such as different risk attributes and making untruthful price revelations through bluffing for improving profits. Market-makers provide a reliable and economic technology for efficient operation of financial markets and we believe that future investigation of their strategies along the directions we explored in this paper will lead to more efficient market performance.

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